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# Non-Local Quantum Theory of the Scalar Field

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**Abstract.** A scheme of the construction of the $S$-matrix according to the perturbation theory which is free of the ultraviolet divergences is suggested by the example of the one-component scalar quantized field. The causality is violated in small space-time region. The effects which are due to the causality violation at large distances are described by very high perturbation orders, and are therefore very small in the framework of the perturbation theory.

## 1. Introduction. The Macrocausality Condition

As is well known, the main trouble in the quantum field theory due to the perturbation expansion of the S-matrix in the usual Lagrangian of the system quantized fields is to remove ultraviolet divergences. Many attempts have been made to overcome this trouble by refusing the locality principle, as has been first suggested by Vatagin [1]. However there are many difficulties in the non-local quantum field theory. It seems to us that at present it is far from being completed[1].

The present paper is also an attempt to remove the ultraviolet divergences by introducting non-locality into the interaction Lagrangian.

One of the main difficulties in constructing the non-local quantum field theory appears to be the formulation of the macrocausality of the $S$-matrix. Though there are intutive considerations that the acausal signal should damp rapidly with increasing time or distance, the requirement on the $S$-matrix behaviour has not yet been formulated sufficiently clearly, as it has been done in the case of the microcausality [3].

It seems to us that a reasonable macrocausality condition imposed on the $S$-matrix would be the following generalization of the microcausality condition [3]. Let $\varphi(x)$ be a field operator. Then the $S$-matrix should satisfy the following condition[2]

$$\frac{\delta}{\delta\varphi(x)}\left(\frac{\delta S}{\delta\varphi(y)}\,S^{-1}\right)=0 \tag{1.1}$$

[1] See a review paper on the non-local quantum field theory [2].

[2] From the drivation of the causality condition in [3] it follows that if $SS^{+}\neq 1$ outside the mass shell in the causality condition there appears $S^{-1}$ rather than $S^{+}$.