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On the Cauchy Problem of the Relativistic Boltzmann Equation*

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Abstract. It is shown that the relativistic Boltzmann equation has a local solution through an initial distribution function, if the scattering cross section is bounded for high energies and if the initial distribution falls off exponentially with the energy.

I. Resume of Relativistic Gas Kinetics and Notation[1]

Within the framework of general relativity, a gravitational field is described by a 4-dimensional manifold M together with a metric $G_{ab}(x)$ of signature $(+1, -1, -1, -1)$. The phase space of a particle is the tangent bundle $T(M)$ of M. If x^a, $(a = 0, 1, 2, 3)$, is a coordinate system in M, and if T is a tangent to M at a point x in the domain of x^a, then T can be written

$$T = p^a \left.\frac{\partial}{\partial x^a}\right|_x$$

and $T \to (x^a, p^a)$ is a coordinate system in $T(M)$, called the coordinate system associated with x^a. The set of all coordinate systems in $T(M)$ so obtained defines on $T(M)$ a differentiable structure which turns $T(M)$ into a differentiable manifold of dimension 8. A particle is represented as a path $(x^a(t), p^a(t))$ in $T(M)$, where $x^a(t)$ describes the position of the particle for the parameter-time t, and $p^a(t)$ is its four-momentum at that instant. If we choose the parameter t such that $\frac{\partial x^a}{\partial t} = p^a$ then the tangent of the phase path of a test particle moving under the influence of an external electromagnetic field $F^a{}_b(x)$ and gravitational field $G_{ab}(x)$ is

$$X = p^a \frac{\partial}{\partial x^a} + \{eF^a{}_b(x)\, p^b - \Gamma^a{}_{bc} p^b p^b\} \frac{\partial}{\partial p^a} \,. \tag{1}$$

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[1] For this paragraph cf. BICHTELER (1965). For different approaches, see CHERNIKOV (1962), TAUBER-WEINBERG (1961), LINDQUIST (1966).