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Attempt of an Axiomatic Foundation of Quantum Mechanics and More General Theories, II

GÜNTHER LUDWIG
Institut für theoretische Physik, Universität Marburg

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Abstract. The consequences of an axiomatic formulation of physical probability fields established in a first paper [1] are investigated in case of a finite dimensional ensemble-space.

It will be shown that the stated number of axioms can be diminished essentially. Further the structure of an ortho-complemented orthomodular lattice for the decision effects (also often called "properties" or still more misunderstandingly "propositions") and the orthoadditivity of the probability measures upon this lattice, both, can be essentially inferred from the axioms 3 and 4, *only*. This seems to give a better comprehension of the lattice structure defined by the decision effects.

Particularly, it is pointed out that no assumption (axiom) concerning the commensurability of two decision effects E_1 E_2 with $E_1 \leqq E_2$ must be made but that this commensurability is a theorem of the theory.

I. Fundamental axioms

Since in a preceding paper [1] we briefly discussed the heuristic aspects having led to the statement of the axioms, these axioms shall be quoted very briefly in this paper and, from the first, will be restricted on the case of a finite dimensional ensemble-space.

We will start from two sets:

Let $\underline{K}$ be the set of all ensembles V,

let $\underline{L}$ be the set of all effects F.

Axiom 1. *Over* $\underline{K} \times \underline{L}$ *(cartesian product) a real-valued function* μ *is defined, satisfying:*

α) $o \leqq \mu(V, F) \leqq 1$,

β) $\mu(V_1, F) = \mu(V_2, F)$ *for all* $F \in \underline{L}$ *implies* $V_1 = V_2$,

γ) $\mu(V, F_1) = \mu(V, F_2)$ *for all* $V \in \underline{K}$ *implies* $F_1 = F_2$,

δ) *for each* V *there exists a* F *with* $\mu(V, F) = 1$,

ε) *there exists a* F *(denoted by* 0*) with* $\mu(V, 0) = o$ *for all* $V \in \underline{K}$.

Definition 1. Let B be the set of all functions $X(F)$ on $\underline{L}$ with

$$X(F) = \sum_{i=1}^{n} a_i \mu(V_i, F), \quad V_i \in \underline{K} \tag{1}$$

a_i real numbers and n any finite integer.