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A Remark on a Theorem of B. Misra

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Abstract. The two sided ideals of the C^*-algebra generated by local v. Neumann algebras are investigated.

I. Introduction

B. Misra [1] has shown that the algebra of all local observables is simple when the following conditions are fulfilled:

1. The algebra is given as a concrete C^*-algebra in a Hilbert space fulfilling the usual assumptions of local ring systems.
2. The rings associated with bounded open regions are v. Neumann algebras.
3. For any bounded open region $\mathcal{O}$ exists another bounded open region $\mathcal{O}_1$ containing $\mathcal{O}$ such that the ring associated $\mathcal{O}_1$ is a factor.

The third condition, however, has not been derived from the other two assumptions even when we assume that the von Neumann algebra generated by the global C^*-algebra is a factor. Since in recent years different representations of the C^*-algebra of all local observables have been discussed [2], [3], [4] it is desirable to have a characterization of all two-sided ideals in the general case where 3. is not assumed. We will show that the theorem of Misra stays true without assuming 3., i.e. the C^*-algebra generated by all local observables is simple if it contains no center. For later use we will also consider some more general algebras.

II. Assumptions and notations

We denote by $\mathcal{O}$ open bounded regions in the Minkowski-space and write:

$\mathcal{O}_1 \times \mathcal{O}_2$ if $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated.

$\mathcal{O}_1 < \mathcal{O}_2$ if $\mathcal{O}_1 \subset \mathcal{O}_2$ and there exists an $\mathcal{O}_3 \subset \mathcal{O}_2$ with $\mathcal{O}_1 \times \mathcal{O}_3$.

$\mathcal{O}_1 \leqslant \mathcal{O}_2$ if there exists a neighbourhood $\mathcal{N}$ of the origin such that $\mathcal{O}_1 + x < \mathcal{O}_2$ for all $x \in \mathcal{N}$.

We denote by a local ring system $\{\mathcal{R}(\mathcal{O})\}$ a family of rings of operators in a fixed Hilbert space $\mathcal{H}$ submitted to the following conditions: