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Uniqueness and Symmetry Breaking in S-Matrix Theory

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Abstract. Assuming that the physical world is a solution of the S matrix equations, nonlinear functional analysis enables its uniqueness to be tested experimentally. As a first step, we develop such tests within the limits of partial wave dispersion relations, *with crossing symmetry included.* They are closely related to Levinson's theorem. We show that they give conditions for the validity of the bootstrap hypothesis, of the dynamical generation of symmetries, and of Dashen-Frautschi perturbation theory. They do not appear to be satisfied experimentally.

1. Introduction

Of the various theoretical frameworks for elementary particle physics, S matrix theory is especially remarkable for its mathematical inadequacy and its experimental success. In part the former is due to the incompleteness of its axioms, about which I have nothing to say. However, even in the case where equations exist and where all the experimental successes have been obtained, most calculations employ very crude approximations. There has been considerable doubt as to whether the deeper predictions of unique sets of self-consistent particles possessing dynamically determined symmetries, do in fact follow from the S matrix equations, and are not just self-consistency conditions on the rough approximations used. In the present work, I want to investigate this question using nonlinear functional analysis.

The equations of S matrix theory are nonlinear integral equations. They define nonlinear operators in a Banach space of physically acceptable scattering amplitudes. The best known results of nonlinear functional analysis are fixed point theorems, which decide whether solutions of such equations exist. However they involve intricate topological questions (and would lead to solipsist titles), so at present I will only consider the easier problem of whether a given solution is unique. Unless S matrix theory is completely wrong, the observed physical universe must be a solution of the S matrix equations. Some people hope it is the only solution. Nonlinear functional analysis lets us test this hope *experimentally.* This is done by Banach space implicit function theorems which tell us, using only experimentally observable quantities, whether the observed universe is an isolated solution of the S matrix equations