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The Energy Momentum Spectrum of Quantum Fields

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Abstract. It is proved, assuming Einstein causality, that the energy-momentum spectrum of a quantum field cannot be bounded. More is known under special assumptions [1, 4]. Our main concern is the method and general applicability of the result.

I. Introduction

The Haag-Araki formulation of local quantum field theory associates with open regions $\mathcal{O}$ of Minkowski space-time R^4 von Neumann algebras $\mathcal{R}(\mathcal{O})$ on a Hilbert space $\mathcal{H}$. The self-adjoint operators in $\mathcal{R}(\mathcal{O})$ correspond to the bounded observables of the field localized in the region $\mathcal{O}$ of space-time. The dynamics and relativistic invariance of the field are expressed in terms of a (strongly-continuous) unitary representation U of the Poincaré group G on $\mathcal{H}$ in such a manner that $U(g)\,\mathcal{R}(\mathcal{O})\,U(g)^{-1} = \mathcal{R}(g(\mathcal{O}))$, where $g(\mathcal{O})$ denotes the transform of the region $\mathcal{O}$ by the (inhomogeneous) Lorentz transformation g of space-time. (This is *covariance* of U and $\mathcal{R}$.) Further assumptions are made — among them:

$\{\mathcal{R}(\mathcal{O}) : \mathcal{O}$ open in $R^4\}$ and $\{\mathcal{R}(\mathcal{O}_s) : \{\mathcal{O}_s\}$ an open covering of $R^4\}$ both generate the same C^*-algebra $\mathfrak{A}$ (the *quasi-local algebra of* the system). (1)

$$\mathcal{R}(\mathcal{O}_1) \subseteqq \mathcal{R}(\mathcal{O}_2)' \text{ if } \mathcal{O}_1 \text{ and } \mathcal{O}_2 \text{ are space-like separated.} \tag{2}$$

$$\mathcal{R}(\mathcal{O}_0) \subseteqq \mathcal{R}(\mathcal{O}) \quad \text{if} \quad \mathcal{O}_0 \subseteqq \mathcal{O}\,. \tag{3}$$

According to the theory of unitary representations of locally compact abelian groups (generalization of Stone's theorem) [3: p. 147] the restriction of U from G to the 4-translation group (the additive group of R^4) gives rise to a projection-valued measure E on the dual $\hat{R}^4$ of R^4, this dual being identified with energy-momentum space, such that $U(a) = \int_{\hat{R}^4} \exp(ia \cdot p)\, dE(p)$. Stone's theorem tells us that each of the one-parameter unitary groups $t \to U(ta)$ has an infinitesimal generator P_a which is a (not necessarily bounded) self-adjoint operator on $\mathcal{H}$. If a is space-like P_a is the momentum observable conjugate to translation in the direction a. If a is a vector along the time axis, the generator H is

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