Commun. math. Phys. 4, 237—257 (1967)

Unitary and Non-Unitary Representations of the Complex Inhomogeneous Lorentz Group*

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Received June 5, 1966

Abstract. We construct unitary and non-unitary representations of the complex inhomogeneous Lorentz group, including all its unitary, irreducible representations. We discuss the decomposition of these representations when they are restricted to the real inhomogeneous Lorentz group. We also discuss the representations of the Poincaré group for which the translation subgroup transforms under a not necessarily unitary representation. We summarize briefly the physical motivation for this study.

I. Introduction

This is the first of (at least) two articles on the structure and possible applications to scattering theory and particle physics of the complex inhomogeneous Lorentz group, CILG. In this article we discuss the representations of the group. We do not restrict ourselves to unitary representations. Although we say nothing about irreducibility or "completeness" of the non-unitary representations, we do find all the unitary, irreducible representations of CILG.

In the next article we plan to discuss some possible applications and the physical interpretation of CILG.

We begin in Section II with a series of definitions, for the purpose of naming the various objects we will construct in later sections. We believe that a comparison of the general structure with the particular examples given, will clarify the constructions for the reader. We then define the CILG and several of its important subgroups in Section III. The method of induced representations, which we will use in finding the representations of CILG, requires us to find the "sesquilinear system" representations of certain subgroups of CILG: the "little" groups $\mathfrak{Z}$ and $SL(2C)$ (Sections IV and V), and the translation group, T (Section VI). Finally, we put the results together to find the representations of CILG in Section VII, and of its physical subgroup $\mathscr{P}$, (the Poincaré group) in Section VIII.

In Section V, which reviews the representation theory of $SL(2C)$, we derive the asymptotic form of a certain operator. In Section VIII, we discuss how representations of $\mathscr{P}$ are contained in the representations

* Supported by the National Science Foundation (NSF-GP-58) while at Brandeis University and by the National Science Foundation, Grant NSF-GP-3465, while at the Courant Institute of Mathematical Sciences.