Commun. math. Phys. 4, 217—236 (1967)

The Representations of the Oscillator Group*

R. F. Streater
Physics Department Imperial College London S.W. 7

Received September 15, 1966

Abstract. Using the Mackey theory of induced representations all the unitary continuous irreducible representations of the 4-dimensional Lie group G generated by the canonical variables and a positive definite quadratic 'hamiltonian' are found. These are shown to be in a one to one correspondence with the orbits under G in the dual space $\underline{G}'$ to the Lie algebra $\underline{G}$ of G, and the representations are obtained from the orbits by inducing from one-dimensional representations provided complex subalgebras are admitted. Thus a construction analogous to that of Kirillov and Bernat gives all the representations of this group.

Introduction

The *general theory of induced representations* as developed by Mackey [1] allows one to classify and explicitly construct all the unitary irreducible continuous representations of any semi-direct product of groups whose projective representations are known, provided the semi-direct product is 'regular'. The criterion of regularity can be examined explicitly in given cases, and holds for many groups of interest, such as the Poincaré group.

By applying Mackey's theory inductivity to nil-potent Lie groups, Kirillov [2] has given a very neat method for finding all the (unitary continuous irreducible) representations of any nil-potent group. This method can be applied to any Lie group, whose structure may not be as simple in terms of semi-direct products as the nil-potent case. The question then arises, does the Kirillov construction give all the representations of a more general group?

For solvable Lie groups, with the extra property of being *exponential,* Bernat [3] has proved that the Kirillov construction does indeed give all the representations (from now on, representations will mean unitary continuous). For compact semi-simple groups, it gives all the representations, and for non-compact semi-simple groups it seems to give many of them. The question arises, for which groups does it give all the representations?

* The research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office Aerospace Research, United States Air Force.