Commun. math. Phys. 4, 203—216 (1967)

Fields at a Point

A. GROSSMANN *

Institut des Hautes Etudes Scientifiques
91. Bures-sur-Yvette — France

Received June 10, 1966

Abstract. Free fields and Wick products without smearing are studied as operators in a nested Hilbert space. It is shown that Wick products are entire analytic in complex space-time and that products of field operators are holomorphic in the forward tube. The Poincaré group is represented by unitary automorphisms of the Fock space.

I. Introduction

A quantized field at a point cannot be a reasonable operator in the Hilbert space of states [1]. It is known, however, that free fields at a point are mappings from a suitable topological vector space into its dual [2]; one expects therefore that they are operators in some nested Hilbert space[1].

It will be shown here that this is indeed so. The construction is uneventful and involves only Hilbert spaces that are very similar to the "physical" one. The norms in these spaces are *not* Lorentz invariant. Nevertheless, the Poincaré group is represented by unitary automorphisms of the "Fock nested Hilbert space".

Some of the results are stated below:

Theorem. *Let X be the positive hyperboloid of mass $M \geqq 0$, and let μ be the Lorentz invariant measure on X. Denote by $H_I^{(1)}(X; \mu)$ the nested Hilbert space corresponding to $(X; \mu)$ (see Section 2a) and by $T_I(X; \mu)$ the tensor algebra over $H_I^{(1)}(X; \mu)$ (see Section 2b). Then:*

(i) *For any ν space-time points $x_1, \ldots, x_\nu$ ($\nu \geqq 1$) the Wick product $:A(x_1) \ldots A(x_\nu):$ belongs to $L(T_I; T_I)$. In particular, the free[2] field operator $A(x)$ belongs to $L(T_I; T_I)$ for every x.*

(ii) *The operator family $:A(x_1) \ldots A(x_\nu):$ is the restriction, to real x, of a family $:A(z_1) \ldots A(z_\nu):$ which is entire analytic (in the sense described in Section 2g) in the arguments $z_1, \ldots z_\nu$.*

(iii) *The product $A(z_1) \ldots A(z_\nu)$ (without Wick ordering) is holomorphic in the domain*

$$\operatorname{Im}(z_j - z_{j-1}) \in V_+ \quad (j = 2, \ldots, \nu)$$

* Present address: Physique Theorique, Université Aix-Marseille, Place Victor Hugo, Marseille.

[1] We shall use the terminology and notation of [3] and [4].

[2] For the sake of simplicity, we consider only neutral scalar fields.