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Homomorphisms and Direct Sums of Nested Hilbert Spaces

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Abstract. We prove some elementary facts about homomorphisms and infinite direct sums of nested Hilbert spaces.

Introduction

In a previous paper [1] we have studied a class of vector spaces endowed with a certain "self-dual" structure. The applications to follow — as well as mathematical decorum — require the introduction of suitable definitions concerning isomorphisms, subspaces, group representations etc.

All these concepts can be obtained in a standard way [2] from that of homomorphism; so our first task is to single out homomorphisms among all operators between nested Hilbert spaces.

For the special case of chains of Hilbert spaces, the natural definition can be found in PALAIS[1] [3]. The generalization to arbitrary nested Hilbert spaces is studied in Sections 2a to 2c below. It is preceded by a discussion of relevant results from [1] (Sections 1a to 1e).

The last part of the paper (Sections 3a to 3b) deals with infinite direct sums of nested Hilbert spaces. The results are used in the accompanying paper which is concerned with quantized fields.

1. Preliminaries

The definition of nested Hilbert space is recalled in Section 1a; that of operator and of adjoint operator in Section 1b. The reader should keep in mind the definition of the set $J(A)$ which describes the "regularity" of the operator A with respect to the space H_I. The larger $J(A)$, the "better" or "smoother" A.

Operators between nested Hilbert spaces behave as bounded operators between Hilbert spaces as far as addition, multiplication by scalars *and the taking of adjoints* go. The product of n operators is defined if and only if the sets $J(A^{(j)})$ $(j = 1, \ldots, n)$ satisfy a certain condition (Section 1c).

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