Commun. math. Phys. 4, 157—176 (1967)

Galilean Quantum Field Theories and a Ghostless Lee Model*

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Abstract. Galilean quantum field theories, i.e. kinematically consistent nonrelativistic quantum theories with an infinite number of degrees of freedom, are considered. These theories transcend the frame of ordinary quantum mechanics by allowing genuine particle production processes to be described. The general structure of such theories is discussed and contrasted with the typical structure of relativistic quantum field theories which they may serve to illustrate a contrario. Despite the mass superselection rule, and due to the weakening of local commutativity conditions, galilean quantum field theories are much less constrained than relativistic ones. The CPT and spin-and-statistics theorems do not hold here, neither does Haag's theorem.

Second-quantized quantum mechanics, some many-body theories (such as the polaron model) and static models are briefly examined, giving simple examples and counterexamples of the general properties asserted.

A Lee model with complete nonrelativistic kinematics is studied and shown to give a consistent non-trivial example of a galilean quantum field theory. In this "GaliLee" model, while all the desirable features of the usual Lee model remain, the ghost problem disappears and the local coupling limit gives meaningful expressions for the physical quantities. The $(V \leftrightarrow N\theta)$ sector is solved for the physical V-particle whose renormalization constant is finite for local coupling, and for the N-θ scattering amplitude, which obeys an exact effective range formula in the same local limit. The elementarity of the V-particle is discussed in relation with the $Z = 0$ rule and Levinson's theorem which is found wanting. The case of an unstable V-particle is also considered, and leads, for local coupling, to an exact Breit-Wigner formula for the N-θ scattering cross-section.

Introduction

In this paper, we wish to discuss and illustrate the concept of galilean quantum field theories, that is consistent nonrelativistic quantum field theories, in agreement with the galilean principle of relativity.

There are, in our opinion, several reasons for such an investigation:

– it is interesting to know if the framework of nonrelativistic quantum theory is limited to the usual quantum mechanics, dealing with fixed numbers of particles, or if it can be enlarged to describe genuine field theories, allowing particle production processes, which usually only appear in relativistic situations. We will show that the second alternative holds.

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* Revised Version. September 1966.

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