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Absence of Interaction as a Consequence of Good Ultraviolet Behavior in the Case of a Local Fermi Field

ROBERT T. POWERS*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

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Abstract. The ICAR theorem asserts that a local relativistic Fermi field is necessarily a free field, if it satisfies the canonical anticommutation relations, irreducibility of the fields at a fixed time, and certain regularity conditions. The regularity conditions are slightly stronger than the requirement that the mass renormalization be finite. It follows that an interacting Fermi field must violate one or more assumptions of the ICAR theorem.

I. The ICAR theorem

We prove a theorem which demonstrates the intrinsic difficulties associated with the construction of a relativistic quantum field theory satisfying an irreducible representation of the canonical anticommutation relations. The theorem will be called the ICAR theorem. The ICAR theorem asserts that a local relativistic Fermi field is necessarily a free field, if it satisfies the canonical anticommutation relations, irreducibility of the fields at a fixed time, and certain regularity conditions. The regularity conditions are slightly stronger than the requirement that the mass renormalization be finite. It follows that an interacting Fermi field must violate one or more assumptions of the ICAR theorem.

A Fermi field will violate the assumptions of the ICAR theorem if either the mass renormalization or field strength renormalization is infinite. It is well known that the perturbation expansion for interacting Fermi fields implies that the mass and field strength renormalizations are infinite. The ICAR theorem suggests that the divergence of the renormalization constants is intrinsic to interacting Fermi fields and is not particular to the perturbation theory approach. This contention is similar to but weaker than KÄLLÉN's contention [1], that one of the renormalization constants in quantum electrodynamics is infinite.

Even if the renormalization constants are finite, a Fermi field may fail to satisfy the assumptions of the ICAR theorem because the fields at a fixed time are reducible. This means there are non-trivial operators which commute with the fields at a fixed time. In this sense the ICAR

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