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Non-Existence of Trouser-Worlds

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Abstract. Simple restrictions are derived on the global structure of normal hyperbolic Riemannian spaces. As a consequence, cosmological models of the trouser type are ruled out irrespective of field equations.

A well-known phenomenon in astronomy is the appearance of new visible objects in the sky. This led Jordan to the conjecture that gravity theory might offer cosmological models in which separate *spaces unite* at some finite time, yielding a discrete growth of the observable matter in the universe. In order to have an imaginable model in mind, think of a 2-dimensional pair of trousers in Minkowskian 3-space such that the vertical direction is timelike: horizontal sections of this "*trouser world*" are two circles near the bottom, and one circle at hip level. In other words: space sections are disconnected at some time, and connected at another. However, closer inspection shows that this model does not carry a regular metric: near the saddle point its signature changes from normal hyperbolic to (positive) definite. Is this failure unavoidable, i.e. independent of imbedding, and dimension? We are going to prove a simple lemma which indeed denies the existence of trouser worlds. (Further relevant global properties will be presented by Seifert in a later article.)

Lemma. *Consider a g-complete* [1] *Riemannian space of continuity class C_2 containing a continuous surface Σ, and a continuous unit vector field u^a over Σ. Follow the geodesics which start on Σ with direction u^a up to (eigen) distance s (from Σ). The "geodesic map" $\Sigma \to \Sigma(s)$ of Σ onto the points $\Sigma(s)$ thus obtained is continuous for every s.*

Proof. It suffices to show that the image points $x_k(s)$ of a converging point sequence x_k in Σ converge towards the image point $x(s)$ of the limit x of x_k (with respect to the coordinate topology). Note that the geodesics of a C_2-space cannot split because their differential equation satisfies a Lipschitz condition. Suppose now that for a certain s, $x_k(s)$ does not converge towards $x(s)$, and call s_0 the infimum of all such s. Consequently, the geodetic segments g_k converge pointwise towards g for all s smaller than s_0; and so do their tangent vectors