Commun. math. Phys. 4, 64—76 (1967)

Fermions and Associated Bosons of One-dimensional Model

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Received July 1, 1966

Abstract. The representation of the canonical commutation relations involved in the construction of boson operators from fermion operators according to the recipe of the neutrino theory of light is studied. Starting from a cyclic Fock-representation for the massless fermions the boson operators are reduced by the spectral projectors of two charge-operators and form an infinite direct sum of cyclic Fock-representations. Kronig's identity expressing the fermion kinetic energy in terms of the boson kinetic energy and the squares of the charge operators is verified as an identity for strictly selfadjoint operators. It provides the key to the solubility of LUTTINGER's model. A simple sufficient condition is given for the unitary equivalence of the representations linked by the canonical transformation which diagonalizes the total Hamiltonian.

1. Indroduction

A number of theoretical physicists, among them DE BROGLIE, JORDAN, BORN and NAGENDRA-NATH, have worked on a neutrino theory of light. In 1938, PRYCE [1] reviewed the results of the efforts up to that time and demonstrated, that in a four-dimensional space-time version of the theory the conditions imposed by the commutation relations for the field amplitudes and the connection between spin and polarization are incompatible. PERKINS [2] in his recent formal attempt avoids this difficulty but does not obtain Bose quantization for the photons.

The two-dimensional variant of the theory, which was originally developed as a proving ground for a realistic theory, has met with some interest in its own right, due to its connection with exactly soluble models. On the one hand, as stressed particularly by WIGHTMAN [3], it is closely related to the THIRRING model, while on the other hand it appears in the context of the soluble quantum mechanical many body problem due to LUTTINGER [4]. MATTIS and LIEB [5] reconsidered this problem and considerably improved the discussion. TOMONAGA [6] did some related work, which was pursued further by ENGELSBERG and VARGA.

* Work supported by the National Science Foundation.