

Derivations and Automorphisms of Operator Algebras

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Abstract. The theorem that each derivation of a C^* -algebra \mathfrak{A} extends to an inner derivation of the weak-operator closure $\varphi(\mathfrak{A})^-$ of \mathfrak{A} in each faithful representation φ of \mathfrak{A} is proved in sketch and used to study the automorphism group of \mathfrak{A} in its norm topology. It is proved that the connected component of the identity ι in this group contains the open ball \mathcal{B} of radius 2 with center ι and that each automorphism in \mathcal{B} extends to an inner automorphism of $\varphi(\mathfrak{A})^-$.

I. Introduction and preliminaries

Our purpose in this paper is to study the group $\alpha(\mathfrak{A})$ of automorphisms of a C^* -algebra \mathfrak{A} together with and in relation to some of its subgroups. We note that the mappings φ of C^* -algebras we consider are assumed to preserve adjoints ($\varphi(A^*) = \varphi(A)^*$) throughout; so that “representation” etc. refer to what is sometimes designated by “*representation” etc. Our particular concern is with $\alpha(\mathfrak{A})$ provided with the topology it acquires from $\mathcal{B}(\mathfrak{A})$, the bounded linear operators on \mathfrak{A} (in its norm), taken in its norm (or, uniform) topology. Recall that each element of $\alpha(\mathfrak{A})$ is an isometry of \mathfrak{A} [10].

In a recent series of papers [16, 18, 24], it is shown that each derivation of a C^* -algebra \mathfrak{A} extends to an inner derivation of the weak-operator closure \mathfrak{A}^- of \mathfrak{A} in every faithful representation of \mathfrak{A} . Each such derivation is a bounded linear operator [23] and, as such, the infinitesimal generator of a norm-continuous, one-parameter group of automorphisms of \mathfrak{A} . The fact that a derivation extends to one which is inner is equivalent to the fact that the automorphisms of the one-parameter group extend to ones which are inner. These considerations as well as an account of the derivation result, for convenience and completeness, are found in § 2.

The main technical result of this study (Theorem 7) is that each automorphism of a C^* -algebra \mathfrak{A} in the *interior* of the ball \mathcal{B} of radius 2 in $\mathcal{B}(\mathfrak{A})$ with center ι , the identity automorphism of \mathfrak{A} , lies on a norm-

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