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Test Function Spaces for Direct Product Representations of the Canonical Commutation Relations

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Abstract. It is shown how the test function spaces for the field operator and its canonical conjugate are determined by a given irreducible direct product representation of the canonical commutation relations. An explicit characterization of the admissible test functions (so that the smeared out field operators are selfadjoint) is given in terms of any one product state of the representation space.

§ 1. Introduction

Regarding myriotic representations of the canonical commutation relations there exists by now — in addition to general statements [1] — a variety of field theoretical models which illustrate their significance.

Examples are the quantum theory of optical coherence [2] (in the general case of infinite average photon number) and methods for the elimination of infrared divergences [3], [4], the infinite free Bose gas of non-zero density [5] and (in the Fermion case) the Haag solution [6] of the BCS-model.

The mathematical properties of a large class of inequivalent representations have recently been studied by Klauder et al. [7]. These "direct product" representations are generated by defining the Weyl operators $e^{-i\pi(g)}$, $e^{i\varphi(f)}$ roughly as follows:

Let

$$e^{i\varphi(f)} = e^{i\Sigma Q_\nu s_\nu} = W(\underline{s})$$

$$e^{-i\pi(g)} = e^{-i\Sigma P_\nu t_\nu} = V(\underline{t})$$

where $f(x) = \Sigma s_\nu h_\nu(x) = (\underline{s}, \underline{h})$, $g(x) = (\underline{t}, \underline{h})$, $\underline{h}$ a complete orthonormal set of functions and where the P_ν, Q_ν act as canonical conjugate operators on the ν^{th} factor of an infinite direct product

$$H = \prod_{\nu=1}^{\infty} \otimes H_\nu$$

of "oscillator" Hilbert spaces H_ν.

H is non-separable [8], the representations so obtained are highly reducible, and a decomposition into irreducible representations can be given.