Commun. math. Phys. 4, 8—21 (1967)

Degenerate Representations of Non-Compact Unitary Groups. II. Continuous Series

J. FISCHER* and R. RĄCZKA**

International Centre for Theoretical Physics, Trieste

Received June 1, 1966

Abstract. Three degenerate principal series of irreducible unitary representations of an arbitrary non-compact unitary group $U(p, q)$ are derived. These series are determined by the eigenvalues of the first and second-order invariant operators, the former having a discrete spectrum and the latter a continuous one. The explicit form of the corresponding harmonic functions is derived and the properties of the continuous representations are discussed.

1. Introduction

In our previous paper [1] we obtained two degenerate principal series, $D_M^L(X_+^{p,q})$ and $D_M^L(X_-^{p,q})$, of irreducible unitary representations of an arbitrary non-compact unitary group $U(p, q)$. These series have been realized in the Hilbert spaces of functions defined in the domains

$$X_+^{p,q} = U(p, q)/U(p-1, q) \quad \text{and} \quad X_-^{p,q} = U(p, q)/U(p, q-1) \qquad (1.1)$$

respectively, which are homogeneous with respect to the action of the $U(p, q)$ group (see [2]). The representation labels M and L determine the eigenvalues, M and λ, of the first and second-order invariant operators $\hat{M}$ and $\Delta(X_\pm^{p,q})$ respectively and both possess a discrete spectrum.

In the present paper we investigate the properties of the continuous series of degenerate representations of the $U(p, q)$ groups which are characterized by continuous values of λ and discrete values of M. We derive three such series of representations, the first two being related to the manifolds $X_+^{p,q}$ and $X_-^{p,q}$ given by (1.1) and the third being related to the manifold

$$X_0^{p,q} = U(p, q)/T^{p+q-2} \boxed{s}\, U(p-1, q-1)\,. \qquad (1.2)$$

Here, T^{p+q-2} is the group of translations in the $(p+q-2)$-dimensional complex space C^{p+q-2} and $\boxed{s}$ means the semidirect product. As will be shown later, the homogeneous spaces $X_\pm^{p,q}$ and $X_0^{p,q}$ can be represented as certain hypersurfaces in the $2(p+q)$-dimensional Minkowski space $M^{2p,2q}$.

---

* On leave of absence from Institute of Physics of the Czechoslovak Academy of Sciences, Prague, Czechoslovakia.

** On leave of absence from Institute of Nuclear Research, Warsaw, Poland.