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Range of Forces and Broken Symmetries in Many-Body Systems*

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Abstract. It is argued that for a many-body system with short range forces the commutators between local operators at different times will be fast decreasing for large spatial separations.

This allows the adaptation of many discussions in relativistic field theories to the case of a many-body system with short range forces. In particular one has the result that a spontaneous breakdown in symmetry implies the existence of excitations of arbitrarily small energy. However this result has essentially only one application: We know that the Galilei invariance is always broken (in a medium of finite density). Therefore one concludes that in a many-body system with short range forces there can never be an energy gap.

I. Introduction

In the general framework of relativistic field theories[1] the postulate of commutativity of local observables for space-like separations plays a very fundamental role. A recent application of this postulate was a proof of Goldstone's theorem [3] which did not involve the existence of covariant field operators [4], [5]. In this connection we were led to search for the behaviour of commutators (anti-commutators) of local quantities at different times in a non-relativistic many-body system.

There one can safely start from canonical equal time (anti) commutation relations and through the equations of motion determine *in principle* the behaviour of (anti) commutators for different times. In practice of course we are not able to solve the equations of motion except in the two extreme cases of weak and strong coupling. The analysis of those two limiting situations in Section II gives us confidence that in general the rate of decrease of the commutator for fixed time difference and large spatial separations will be closely related to the rate of decrease of the interparticle potential.

Since the discussion of Goldstone's theorem in [4] could have been made by replacing the postulate of strict local-commutativity by the weakened assumption

$$\lim_{|\mathrm{x}|\to\infty} \|[A(\mathrm{x}, t), B]\| \, |\mathrm{x}|^n = 0 \tag{1}$$

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[1] See for example [1], [2].