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Current Commutation Relations in the Framework of General Quantum Field Theory

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Abstract. In this paper we give a rigorous formulation of Gell-Mann's equal time commutation relations in the framework of general quantum field theory. We show that this can be achieved despite the nonexistence of charge operators for non-conserved currents. Starting from the properly formulated equal time commutation relations of "generalized charges", we justify the application of the Gauss-Theorem and we discuss the limits for large times of time dependent "generalized charges". The Jost-Lehmann-Dyson representation is used in order to show that the equal time commutation relations always lead to exactly one, frame independent, sum rule. We discuss the connection between properties of the Jost-Lehmann-Dyson spectral function and the convergence of Adler-Weisberger type sum rules.

1. Introductory remarks

The most convincing success of the equal time commutation relations between vector and axial vector currents originally proposed by Gell-Mann [1] is the derivation of sum rules of the Adler-Weisberger type [2], [3]. In the original presentation of Adler and Weisberger this derivation was very involved and the independence on the frame of reference was not shown. Fubini and Furlan [4] proposed subsequently a simpler and aesthetically more appealing covariant method based on the commutation relations of "generalized charges"; however, the weak point of their derivation was the discussion of the boundary terms for large times and the ambiguity due to the bad large distance behaviour of the retarded matrix-element. Furthermore, it was not realized, that retardation i.e. the multiplication with the step function $\Theta(x_0)$ does not introduce any ambiguous subtraction constant. This follows from a more careful formulation of the commutation relations which takes into account the distribution theoretical aspects. The use of Gauss's theorem for field operators and a careful computation of time limits will resolve the ambiguities for low energies (i.e. the intermediate one particle ambiguity).

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