

Discrete Degenerate Representations of Non-Compact Unitary Groups

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Abstract. Two degenerate principal series of irreducible unitary representations of an arbitrary non-compact unitary group $U(p, q)$ are derived. These series are determined by the eigenvalues of the first and second-order invariant operators, which are shown to possess a discrete spectrum. The explicit form of the corresponding harmonic functions is derived and the properties of the discrete representations are discussed in detail. Moreover, in the Appendix, we derive the properties of the corresponding degenerate representations of an arbitrary compact $U(p)$ group.

1. Introduction

Irreducible unitary representations of semi-simple Lie groups can be constructed in the Hilbert space $\mathcal{H}(X_i)$ of the functions defined in a domain X_i , which is some homogeneous space of the type

$$X_i = G/G_i, \tag{1.1}$$

G_i being a closed subgroup of G . In the case of the non-compact unimodular unitary groups $SU(p, q)$ we can construct two classes of homogeneous spaces having a compact and non-compact stability group, respectively. The first class contains the following symmetric Cartan spaces (see [1], Chap. IX):

Table

X	rank	dimension of X
$SU(n)/SO(n)$	$n - 1$	$\frac{1}{2}(n - 1)(n + 2)$
$SU(2n)/Sp(n)$	$n - 1$	$(n - 1)(2n + 1)$
$SU(p + q)/S[U(p) \times U(q)]$	$\min(p, q)$	$2pq$
$SU(p, q)/S[U(p) \times U(q)]$	$\min(p, q)$	$2pq$

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