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Condensation of Lattice Gases

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Abstract. Techniques due to R. L. DOBRUSHIN and R. GRIFFITHS are combined to prove the existence of a first order phase transition at low temperature for a class of lattice systems with non nearest-neighbour interaction.

1. Introduction

In recent papers, DOBRUSHIN [2] and GRIFFITHS [5] have proved that a gas with nearest-neighbour attractive interaction on a cubic lattice in ν dimensions ($\nu \geqq 2$) undergoes a first order phase transition. DOBRUSHIN and GRIFFITHS compute explicitly a region where two phases coexist and the pressure is a constant function of density at constant temperature.

While the result and techniques used are not quite new (see [7], [9], [10]), they are important in giving a simple model for proofs of condensation[1]. In this note we shall combine the techniques of DOBRUSHIN and GRIFFITHS (these authors worked independently) to prove the existence of a first order phase transition at low temperature for a class of lattice systems with non nearest-neighbour interaction. Our main result is the theorem of Section 3, which the reader may consult at this point. Section 2 contains preparatory material for the proof of the theorem.

2. Systems with pair interactions on a lattice

We collect in this section some definitions and known results.

We consider a ν-dimensional lattice with vertices $\mathbf{k} = (k^1, \ldots, k^\nu)$ where $k^1, \ldots, k^\nu$ are integers. Particles on the lattice are assumed to interact through a pair potential Φ such that $\Phi(\mathbf{k}) = \Phi(-\mathbf{k})$ and

$$\Phi(0) = +\infty, \sum_{k \neq 0} |\Phi(\mathbf{k})| = D < +\infty . \tag{2.1}$$

[1] One of us (D. R.) has been informed by V. ARNOLD and R. BALESCU that further results in this direction have been obtained by SINAI and BEREZIN; on the other hand DOBRUSHIN has extended his results to certain lattice gases with non nearest neighbour interaction (private communication).