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States of Physical Systems

David Ruelle

Institut des Hautes Etudes Scientifiques
91 Bures-sur-Yvette — France

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Abstract. States of physical systems may be represented by states on B^*-algebras, satisfying certain requirements of physical origin. We discuss such requirements as are associated with the presence of unbounded observables or invariance under a group. It is possible in certain cases to obtain a unique decomposition of states invariant under a group into extremal invariant states. Our main results is such a decomposition theorem when the group is the translation group in ν dimensions and the B^*-algebra satisfies a certain locality condition. An application of this theorem is made to representations of the canonical anticommutation relations.

1. Introduction: B^*-algebras and states

The main purpose of this paper is to prove a theorem yielding an integral representation of invariant states on a B^*-algebra in terms of extremal invariant states. The theorem and related results are presented in Section 3 to which the reader may proceed directly[1]. This first and the second sections are devoted to motivation and some background information. Sections 4 and 5 contain the proof of the theorem of Section 3 and Section 6 an application to canonical anticommutation relations. Other applications, to the states of equilibrium statistical mechanics, will be presented in a forthcoming paper.

The use of C^*-algebras in physics, proposed by Segal and Haag, has been mostly restricted to the study of canonical commutation relations and field theory. Other domains, like statistical mechanics, are however potential fields of application.

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[1] After a first version of this paper was completed, I benefited from conversations with Kastler and Robinson. These authors and Doplicher ([4], Section 5) have obtained, independently, results corresponding roughly to Corollary 2, Section 3, of the present paper. Furthermore, Robinson [6] has obtained important generalizations of Lemma 4, Section 4, and Corollaries 1 and 2, section 3. Contrary to what is done here, Robinson makes systematic use of Hilbert space methods. I am greatly indebted to Kastler and Robinson for discussing with me their results, a large part of which is not yet written down [6]. These discussions have prompted me to make a few changes to the original version of this paper, notably by replacing "local" by "asymptotically Abelian" [4] B^*-algebras and appending two remarks (after the theorem in Section 3 and after Lemma 4, Section 4) which relate the present work to the forthcoming paper [6] of Kastler and Robinson.