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Rigged Hilbert Spaces in Quantum Mechanics*

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Abstract. It is shown how rigged Hilbert spaces may be constructed in quantum mechanics, and the properties of the resulting spaces are derived. The theory is applied to non-relativistic quantum systems of n interacting particles. The spectral theory in rigged Hilbert spaces is developed and the results necessary for the application to the Dirac formalism are derived.

I. Introduction

It has been shown in a previous paper [1] how the concept of a rigged Hilbert space provides a natural framework for introducing the Dirac bra and ket spaces into quantum mechanics. It was further shown how the introduction of extra structure into the Hilbert space by labelling certain of the observables not only provided a description of the differences between different physical systems, but also allowed a natural construction of a rigged Hilbert space associated with that quantum system. It is the purpose of this paper to go into the details of this construction and to deduce certain general topological properties of the spaces constructed in this way. To this end the actual interpretation in terms of labelled observables is irrelevant, and it is sufficient to have a suitable set of distinguished operators on Hilbert space, which are to be made continuous. This abstract theory is applied to the non-relativistic theory of n interacting particles to derive necessary and sufficient conditions on the potential, so that a rigged Hilbert space may be constructed by applying the canonical method.

As the descriptions of the spectral theory in rigged Hilbert spaces are rather scattered throughout the mathematical literature and no one source gives a description which is reasonably elementary and of sufficient generality, it seemed worthwhile devoting the second part of this paper to a brief exposition of this theory. The theory is presented in the context of a spectral theory for semiinner product spaces as this makes for little additional complication and would in any case be necessary for

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