Commun. math. Phys. 3, 75—97 (1966)

# Analytic Continuation of Group Representations. III*

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Received April 22, 1966

**Abstract.** The connection between the ideas of "contraction" and "analytic continuation" of Lie algebras and their representations is discussed, with particular emphasis on the contraction of the Poincaré to the Galilean group.

## 1. Introduction

We continue the study of the relation between analytic continuation of Lie algebras, their representations and Lie algebra cohomology. The first topic we will treat will be a further development of the formalism when a Lie algebra structure is fixed, and an irreducible representation of it is analytically continued. In [4] we showed that, if the relevant first cohomology group vanishes, then the Casimir operators of the Lie algebra are constants of the deformation parameter. Here, we will study the formalism for the case where the first cohomology group does not vanish. We will obtain some insight into one of the main problems, namely, discovering when the first cohomology group is finite dimensional.

Our next topic will be to continue both the Lie algebra structure and the representation. This will provide a tie-up between Lie algebra cohomology theory and the Gell-Mann formula for the representations of Lie algebras. Again, we will find that the beautiful ideas of the Kodaira-Spencer theory of deformation of structure provide us with a deep insight into the already known situation, and should be an invaluable guide to extending the existing theory to new situations. The case of the contraction of the Poincaré to the Galilean group will be treated in some detail.

I would like to thank R. KALMAN for his hospitality at Stanford University while this paper was written.

## 2. The effect of continuation of representations on the universal enveloping algebra

Let G be a Lie algebra, with $X$, $Y$, . . . denoting its typical elements, $[X, Y]$ its bracket. Recall that $U(G)$, *the universal associative enveloping algebra* of $G$ is defined in the following way [3, 5]:

* This research was supported in part by the Office of Air Force Scientific Research AF 49 (638)-1440.