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Analytic Continuation of Group Representations II*

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Abstract. The paper deals with the general background connecting the ideas of analytic continuation and contraction of Lie algebras and their representations. A connection is also established between the Kodaira-Spencer deformation theory and the theory of cohomology of Lie algebras.

1. Introduction

In this paper, we follow up some relations discovered in [1] between the contraction of a Lie algebra and the various "analytic continuations" possible for its skew-Hermitian representations. Consider the Lie algebra of $SL(2, R)$. One contraction leads to the group of rigid motions in the plane. Conversely, through an "expansion" process (using the "Gell-Mann formula") a representation of the latter group gives a one parameter family of representations of $SL(2, R)$, which is just the familiar "continuous series". Another contraction leads to the Heisenberg Lie algebra, i.e. to that generated by the Heisenberg commutation relations $[p, q] = 1$, $0 = [1, p] = [1, q]$. DOTHAN has pointed out that there is an opposite "expansion" process. (It will be presented in this paper.) It has a new feature: Although the formulas depend continuously on the parameter, only for discrete values of this parameter do the formulas give genuine non-singular operators. The corresponding series of representations of $SL(2, R)$ is the "discrete series", while the associated "physics" is that of the one dimensional harmonic oscillator.

We hope that ultimately these facts will be generalized to the other semisimple groups. The intuitive geometric picture at which we are aiming is that of the equivalence classes of unitary representations of a given Lie group forming a non-compact, finite dimensional space, with the representations of the various contractions of the given group lying on the boundary as "points at infinity".

In working on this program, it will be useful to have the general ideas of "deformation" of Lie algebras and their representations at hand. This fits into a much more general pattern of mathematical thought, first created by K. KODAIRA and D. C. SPENCER, and carried further by many others, e.g., E. CALABI, M. GERSTENHABER, P. GRIFFITHS, M.

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