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Analytic Properties of Scattering Amplitudes in two Variables in General Quantum Field Theory

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Abstract. It is shown that scattering amplitudes have analytic properties as functions of momentum transfer not only for physical, but also for complex energies. This follows from local relativistic field theory for all reactions for which an ordinary dispersion relation can be proved.

It is further shown that such amplitudes are boundary values of analytic functions of the two variables energy and momentum transfer. A domain of holomorphy — which is however not best possible — is given explicitly. It follows then that partial waves can be analytically continued to complex energies.

§ 1. Introduction

As is well known, analytic properties of scattering amplitudes as functions of energy or momentum transfer can be deduced from local relativistic field theory provided that certain mass-spectrum conditions are satisfied. HEPP [1] has given a complete mathematical derivation of these properties. Much less is known in this framework about analyticity in both variables simultaneously. MANDELSTAM [2] obtained a first result of this type. His method is applicable only in cases where ordinary dispersion relations can be derived for all three channels of a reaction. This includes $\pi - \pi$ and $\pi - K$ scattering [3] but not $\pi - N$ scattering. More recently, BROS, EPSTEIN and GLASER [4], [5] proved that for a general two-particle reaction (excluding zero-mass particles) the amplitude is the boundary value of an analytic function of two variables which is holomorphic in a complex neighborhood of all physical points and the different channels are related by crossing-symmetry.

In the following we determine explicitly a complex domain in which the analytically continued amplitude is holomorphic in both variables for any scattering process for which an ordinary (energy) dispersion relation follows from field theory. We show further that this implies some analytic structure of the physical partial waves. These results are obtained by

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