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Proof of the Bogoliubov-Parasiuk Theorem on Renormalization

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Abstract. A new proof is given that the subtraction rules of BOGOLIUBOV and PARASIUK lead to well-defined renormalized Green's distributions. A collection of the counter-terms in "trees" removes the difficulties with overlapping divergences and allows fairly simple estimates and closed expressions for renormalized Feynman integrals. The renormalization procedure, which also applies to conventionally non-renormalizable theories, is illustrated in the φ^4-theory.

1. Introduction

Renormalization in Lagrangian quantum field theory is in the interpretation of BOGOLIUBOV and PARASIUK [1], [2] the extension of certain linear functionals, defined on a subspace of $\mathscr{S}(R^{4n})$, to tempered distributions in $\mathscr{S}'(R^{4n})$ [3]. For instance, the Gell-Mann Low perturbation expansion of the truncated time-ordered distributions has the form

$$\langle T\,\varphi_1(x_1)\ldots\varphi_m(x_m)\rangle^T = \sum_{n=m}^{\infty}\frac{(-i)^{n-m}}{(n-m)!}\int dx_{m+1}\ldots dx_n \times \\ \times\langle T\,\varphi_1^{\mathrm{I}}(x_1)\ldots\varphi_m^{\mathrm{I}}(x_m)\,\mathscr{H}^{\mathrm{I}}(x_{m+1})\ldots\mathscr{H}^{\mathrm{I}}(x_n)\rangle^T . \tag{1.1}$$

Here the truncated vacuum expectation values $\langle\varphi_1^{\mathrm{I}}(x_1)\ldots\mathscr{H}^{\mathrm{I}}(x_n)\rangle^T$ are well-defined for $\mathscr{H}^{\mathrm{I}}(x)$, which are WICK polynomials of the free fields $\varphi_i^{\mathrm{I}}(x)$. On the other hand the straightforward construction of $\langle T\,\varphi_1^{\mathrm{I}}(x_1)\ldots\mathscr{H}^{\mathrm{I}}(x_n)\rangle^T$ by WICK's theorem [4] leads to a product of distributions

$$\prod_{l\in\mathscr{L}}\Delta_l^F(x_{i_l}-x_{f_l}) . \tag{1.2}$$

Formula (1.2) is in general not meaningful as one sees from the definition of Δ_l^F in p-space

$$\tilde{\Delta}_l^F(p) = \lim_{\varepsilon\downarrow 0} i\,P_l(p)\,(p^2 - m_l^2 + i\,\varepsilon)^{-1} , \tag{1.3}$$

where $P_l(p)$ is a polynomial and where $m_l > 0$ is always assumed. Then the convolutions in p-space corresponding to (1.2) can lead to "ultraviolet divergences".

Nevertheless the product (1.2) taken with regularized [5] propagators is a good starting point for the definition of $\langle T\,\varphi_1^{\mathrm{I}}(x_1)\ldots\mathscr{H}^{\mathrm{I}}(x_n)\rangle^T$.

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