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The Schrödinger Equation for Quantum Fields with Nonlinear Nonlocal Scattering*

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Abstract. This paper considers perturbations $H = H_0 + \varepsilon V$ of the Hamiltonian operator H_0 of a free scalar Boson field. V is a polynomial in the annihilation creation operators. Terms of any order are allowed in V, but point interactions, such as $\int :\theta(x)^4: dx$, are not considered. Unnormalized solutions for the Schrödinger equation are found. For $\varepsilon \to 0$, these solutions have a partial asymptotic expansion in powers of ε. The set of all possible pertubation terms V forms a Lie algebra. General properties of this Lie algebra are investigated.

§ 1. Introduction

We consider Hamiltonian operators of the form

$$H = H_0 + V \tag{1.1}$$

where H_0 is the Hamiltonian for a free field and V is a polynomial in the creation annihilation operators $A^{\pm}$. By this we mean that V is a finite sum of monomials V_{lm} of the form

$$V_{lm} = \int A^+(k_1) \ldots A^+(k_l) v_{lm}(k, k') A^-(k'_1) \ldots A^-(k'_m) dk\, dk' \,. \tag{1.2}$$

We require the kernel v_{lm} to be smooth, for example to be in a Schwartz space $\mathscr{S}$. This paper is partly directed toward studying the Lie algebra formed by such H, and it is partly directed toward solving the Schrödinger equation

$$i \frac{\partial}{\partial t} \Psi = H\Psi \,. \tag{1.3}$$

We solve (1.3) for quite general V of the above form. (See Theorems 7.3 and 9.1.) We find in § 7 a preliminary operator T which intertwines H and H_0,

$$H T = T H_0 \,. \tag{1.4}$$

Then

$$T \exp(-it H_0) \Phi(0) = \Psi(t)$$

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