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Analytic Continuation of Group Representations

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Abstract. Some general methods for analytically continuing group representations are presented. In favorable cases, this enables one to recognize the generalization of the "principal", "discrete" and "supplementary" series.

1. Introduction

In [4] we pointed out there is a natural way of "analytically continuing" certain types of unitary representations of non-compact semi-simple Lie groups to finite dimensional representations, i.e. to unitary representations of the compact real form of the group. This phenomenon is quite well-known in case the group is $SL(2, R)$, and is associated in that case with the "Regge pole" ideas of elementary particle physics. Y. NE'EMAN has suggested that the extension of this idea to the other Lie groups used to classify elementary particles (e.g. $SU(3)$) might have physical applications. With these possibilities in the background, this paper is devoted to the mathematical problems. In particular, we will extend the method used in [4] for analytic continuation (which required that the representation satisfied the "Gell-Mann formula") to considerably more general and useful situations. On the other hand, in order to make the methods understandable to a non-specialist, we have not tried to push them to their natural limits.

Let $\mathfrak{G}$ be a Lie algebra, and let H be a Hilbert space. Denote the abstract typical elements of $\mathfrak{G}$ by X, Y, etc. Suppose that $X \to X'$ defines a representation of $\mathfrak{G}$ by skew-Hermitian operators on H. An *analytic continuation* of this representation is a family G^λ of representations $X \to X^\lambda$ of $\mathfrak{G}$ by operators depending complex-analytically on the parameter λ, reducing to the given one for $\lambda = 0$. In this paper, we will consider such continuations defined by "infinitesimal multipliers". These, will be shown to be skew-Hermitian for pure imaginary values of λ, which will define the analog of the "principal series" known for $SL(2, R)$ and other classical groups, [1, 2]. In [4] we showed that a special type of such analytic continuations is reducible for a discrete set of real values of λ. Here, we shall analyze the conditions for such behavior more

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