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A Theorem on Canonical Commutation and Anticommutation Relations

G.-F. DELL'ANTONIO*
Institut des Hautes Etudes Scientifiques

S. DOPLICHER
Istituto di Fisica dell'Università, Roma
and Institut des Hautes Etudes Scientifiques

D. RUELLE
Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, (S. et O.) France

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Abstract. The aim of this note is to characterize representations of the canonical commutation or anticommutation relations which, on a subspace of the "space of test-functions", reduce to a sum of copies of the Fock representation.

1. Generalities[1]

Let $\mathscr{L}$ be a real separated prehilbert space. We assume that $\mathscr{L}$ is separable. One may in a standard way construct a complex Hilbert space $\mathscr{H}$ (Fock space) and, for each $f \in \mathscr{L}$, operators $a(f)$, $a^*(f)$ forming the Fock representation of the canonical commutation relations (CCR) or anticommutation relations (CAR) of $\mathscr{L}$.

In the case of the CAR the operators $a(f)$, $a^*(f)$ are bounded and the C^*-algebra $\mathfrak{A}$ associated with the Fock representation of the CAR is defined as the uniform closure of the algebra generated by all operators $a(f), a^*(f)$. In the case of the CCR the operators $\varphi(f) = \frac{1}{\sqrt{2}}(a(f) + a^*(f))$ and $\pi(f) = \frac{1}{i\sqrt{2}}(a(f) - a^*(f))$ are self-adjoint and one may define the Weyl operators $U(f) = \exp(i\varphi(f))$, $V(f) = \exp(i\pi(f))$. The C^*-algebra $\mathfrak{A}$ associated with the Fock representation of the CCR is defined as the uniform closure of the algebra generated by all operators $U(f)$, $V(f)$. $\mathfrak{A}$ is irreducible and contains the identity operator $\mathbf{1}$ of $\mathscr{H}$.

A (CCR or CAR) representation of $\mathscr{L}$ in a complex Hilbert space $\mathfrak{H}$ is defined by a $*$-homomorphism γ of $\mathfrak{A}$ into the bounded operators on $\mathfrak{H}$ such that $\gamma(\mathbf{1})$ is the identity on $\mathfrak{H}$ and, in the case of the CCR the

* Present address: Courant Institute of Mathematical Science, N. Y. U.

[1] For a general description of CCR and CAR see GÅRDING and WIGHTMAN [4]; for CCR see LEW [5] and references given there to earlier work, in particular by SEGAL; for C^*-algebras see DIXMIER [3].