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On Quadratic Lagrangians in General Relativity

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Abstract. Theories of gravitation similar to General Relativity but with an additional R^2 term in the Lagrangian are explored. The Schwarzschild metric is not the exterior solution that can be continued to the interior of the body to give a positive definite mass distribution. The experimental consequences of R^2 terms are investigated. Furthermore, it is shown that a theory with an R^2 term only possesses an interesting singular dependence on the coupling constant.

1. Introduction

One of the requirements Einstein made in deriving the equations of General Relativity was that the field equations should contain only second derivatives and these only linearly. Postulating this and a coupling of the energy-momentum tensor to the metric of a Riemannian space time, EINSTEIN was led almost uniquely to the famous field equations of General Relativity:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \cdot R = -\varkappa \cdot T_{\mu\nu} \tag{1}$$

derivable from an action integral

$$W = \int d^4x (\sqrt{-g} \cdot R + 2\varkappa \cdot \Lambda(x)) \tag{2}$$

where $\Lambda(x)$ is the action integral of the matter fields.

From Quantum Electrodynamics we know that the vacuum polarization introduces nonlinear terms into the originally linear equations of this theory. There, the classical Lagrangian

$$L = \frac{1}{2} (\mathbf{E}^2 - \mathbf{H}^2) \tag{3}$$

has to be modified to

$$L = \frac{1}{2} (\mathbf{E}^2 - \mathbf{H}^2) + \frac{2\alpha^2}{45 m^4} [(\mathbf{E}^2 - \mathbf{H}^2)^2 + 7(\mathbf{E} \cdot \mathbf{H})^2] \tag{4}$$

(α is the fine-structure constant and m the electron mass, we put $c = 1$) to account for the effects of vacuum polarization in the first non-vanishing order in the coupling constant.

The same phenomenon is likely to occur in the quantisation of General Relativity, although the field equations are nonlinear in this