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Møller Operators for Scattering on Singular Potentials

J. KUPSCH and W. SANDHAS
Physikalisches Institut der Universität Bonn

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Abstract. The existence of Møller operators is proved for singular potentials which decrease more rapidly at infinity than the Coulomb potential. The question of their uniqueness is discussed.

1. Introduction

In the formal theory of scattering processes it is of fundamental interest to investigate whether uniquely defined Møller operators exist. Therefore several authors have dealt with this question, especially for the case of potential scattering. COOK [1] was able to show the existence of uniquely defined Møller operators, as long as the potential $V(\boldsymbol{r})$ is square integrable. This proof was extended by HACK [2] to locally square integrable potentials which at infinity decrease faster than const $\cdot\, r^{-1-\varepsilon}$, i.e., more rapidly than the Coulomb potential, and by KURODA [3] to a class of potentials which includes the cases investigated by COOK and HACK (compare also JAUCH and ZINNES [4]). The physically important points of these considerations are explained in the review article of BRENIG and HAAG [5].

In all the above mentioned proofs it is required that $V(\boldsymbol{r})$ be locally square integrable. For other potentials which do not fulfill this requirement (in the following we will call them "singular"), scattering theory recently has been developed more extensively. Most of these investigations were based on a discussion of the radial Schrödinger equation, i.e., the ordinary methods of stationary scattering theory were used. Although several of the peculiarities connected with singular potentials have been clarified by this procedure, a study from the more transparent viewpoint of formal scattering theory seems desirable. As to the existence of uniquely defined Møller operators some results for spherically symmetric potentials have already been found in this framework. GREEN and LANFORD [6] have shown the existence of Møller operators for potentials which are less singular than $r^{-2+\varepsilon}$ at the origin and LIMIĆ [7] got the same result for highly singular potentials that are repulsive at the origin.

In the following we shall give a relatively simple proof for the existence of Møller operators, also valid for highly singular not spheri-