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Conserved Currents and Associated Symmetries; Goldstone's Theorem*

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Abstract. We consider, in the framework of local field theory with translation symmetry, automorphisms connected with locally conserved currents. We show that such automorphisms lead to symmetries, i.e. are implementable by unitary operators, whenever the smallest mass in the theory is non-zero. Therefore we conclude that a "spontaneously broken symmetry" is possible only in the event that the smallest mass is zero. This establishes the theorem first conjectured by GOLDSTONE.

I. Introduction

A great deal of attention has been recently focused upon the theory of symmetries and "broken symmetries" of physical theories, not only in the realm of elementary particles physics but also in the realm of solid state physics and the many-body problem. As the terms "symmetry" and "broken symmetry" have been used in various senses, let us immediately describe what we understand by this terminology.

All physical theories, such as quantum mechanics, field theory, etc. are at their most basic level algebraic and at this level all physical quantities described by the theory form a *-algebra $\mathfrak{A}$. In general there exist isomorphisms, or automorphisms, of the algebra i.e. mappings of the algebra into itself $\mathfrak{A} \to \mathfrak{A}$, and these automorphisms form the basis of the theory of symmetries. Although physical theories may be dealt with algebraically it is more usual to work at the representation level. Instead of considering the algebra $\mathfrak{A}$ a representation of $\mathfrak{A}$ by operators acting in a Hilbert space $\mathfrak{H}$ is considered and in all theories which are not purely statistical the extra proviso is normally made that the representation should be irreducible. This specialization to a representation introduces a distinction between the various possible kinds of automorphisms

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