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Representations of Groups Containing the Poincaré Group. I*

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Abstract. It is proved that in certain representations of the Poincaré group the mass operator must be identically zero.

1. Introduction

The aim of this series of papers is to study various questions related to the subject of the title, motivated in part by possible relevance to the application of group-theoretical ideas in elementary particle physics. Much of the work will be a continuation of that in [2].

Our immediate starting point is O'Raifeartaigh's theorem [5]. Let P be the Poincaré (= inhomogeneous Lorentz) group, and let G be a group containing P as a subgroup. Consider an irreducible linear representation of G in which the "mass" Casimir operator, $\square$, of $\mathbf{P}$[1] is Hermitian. O'RAIFEARTAIGH shows that no "mass splittings" are possible if G is finite dimensional, i. e. if $\square$ has one discrete eigenvalue m^2, then $\square(\psi) = m^2\psi$ for all vector ψ.

This remarkable property of the Poincaré group has both negative and positive implications for physics. On the negative side, it indicates that many of the recent attempts to put particles of different masses into supermultiplets by a simple group-theoretic scheme are doomed to failure. Positively, it suggests a more elaborate scheme, whereby all particles of a certain type without interactions have the same mass, but the observed mass differences are the result of perturbations due to interactions.

The main purpose of this paper is to indicate further conditions on G which imply that the eigenvalue m^2 must be zero. Roughly, this involves conditions on the normalizer of $\mathbf{T}$ in $\mathbf{G}$. (T is the group of trans-

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[1] The Lie algebra of a group will be denoted by the same letter in bold-faced type. For the notations used here, see [2].