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Some Remarks about the Localization of States in a Quantum Field Theory*

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Abstract. For the case of a field theory with a nuclear space of test functions (for instance, the space of strongly decreasing test functions) compact sets of states are constructed; these correspond to sets of localized states. Only such states are considered which are elements of a fixed subspace of the entire Hilbert space. This subspace belongs to the m-point functions of order less than a certain fixed $2n$.

1. Introduction

If one compares the possibilities of a pure scattering theory with those of a quantum field theory, one is inclined to suppose that the methods of a quantum field theory are more adapted to express localized quantities. It is not quite obvious that a quantum field theory can indeed perform this. It is well known that one can construct in a relativistic quantum field theory dense subspaces of the whole Hilbert space by applying only very restricted subalgebras generated by the field operators to the cyclic vacuum-state. For example, the algebra with test functions all lying in a small neighborhood of a fixed point of four-dimensional space leads to a dense subspace of $\mathfrak{H}$.

It is the intention of the following remarks to get a preliminary idea about the possibilities of the localization of states in a quantum field theory. In this we are guided, roughly speaking, by the concept that two states which are localized at a certain time in two non-overlapping regions of three-dimensional space should be orthogonal or nearly orthogonal to each other [1]. Another approach to the problem of localization is that of KNIGHT [2] and LICHT [3] ("strict-localization"), who begin with the concept of localized observables.

As it may be more interesting to clarify the problem in the relativistic case, we will make the approach in a theory which essentially fulfills WIGHTMAN's axioms [4]. But not all the axioms are needed for the proof

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