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A Proof of the Crossing Property for Two-Particle Amplitudes in General Quantum Field Theory

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Abstract. In the framework of the $\mathscr{L}.\mathscr{S}.\mathscr{Z}.$ formalism, the crossing property is proved on the mass shell for amplitudes involving two incoming and two outgoing stable particles with arbitrary masses. Any couple of physical regions in the (s, t, u) plane corresponding to crossed processes are shown to be connected by a certain domain of analyticity. For every negative value of t, the amplitude is analytic in the cut s-plane outside of a large circle.

1. Introduction

In this paper we propose to prove the property of "crossing" for the scattering amplitudes involving four *stable* particles $A_j (j = 0, 1, 2, 3)$ with arbitrary non-zero masses m_j, within the framework of the $\mathscr{L}.\mathscr{S}.\mathscr{Z}.$ formalism [1].

In this formalism the amplitudes corresponding to the processes

$$A_1 + A_2 \to A_3 + A_0 \quad \text{(and C.T.P.)} \tag{1}$$

$$A_1 + A_3 \to \bar{A}_2 + A_0 \quad \text{(and C.T.P.)} \tag{2}$$

$$\bar{A}_3 + A_2 \to A_1 + A_0 \quad \text{(and C.T.P.)} \tag{3}$$

are different boundary values of a holomorphic function $H(k)$ of the set of complex four-vectors

$$k = \left\{ k_0, k_1, k_2, k_3 \;\Big|\; \sum_{j=0} k_j = 0 \right\} \qquad H(k) \text{ is defined}$$

and analytic in a certain *primitive domain*** Δ in $\mathbb{C}^{12}$. More precisely if P_j is the four-momentum of the particle A_j (or $\bar{A}_j$) $(P_j \in V^+$, for $j = 0, 1, 2, 3)$ and

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** For a description of this domain and for relevant references, we refer the reader to a previous paper [2], the notations of which will also be used in the present paper.