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On the Structure of the von Neumann Algebras Generated by Local Functions of the Free Bose Field

By

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Abstract. It is shown that the von Neumann algebra $R_{\mathfrak{B}}(B)$ generated by any scalar local function $B(x)$ of the free field $A_0(x)$ is equal either to $R_{\mathfrak{B}}(A_0)$ or to $R_{\mathfrak{B}}(:A_0^2:)$. The latter statement holds if the state space $\mathfrak{H}_B$ obtained from the vacuum state by repeated application of $B(x)$ is orthogonal to the one particle subspace. In the proof of these statements, space-time limiting techniques are used.

§ 1. Introduction

Von Neumann algebras of local observables have been introduced into relativistic quantum theory by R. HAAG [1]. A detailed study was made notably by H. ARAKI [2] in a series of papers. ARAKI has shown that most of the rigorous results of general quantum field theory can be obtained in the framework of local observables.

One motivation for introducing these objects is therefore similar to the motivation of introducing algebraic concepts into ordinary quantum mechanics, namely to investigate general structural properties. In quantum mechanics, these algebraic concepts are not of much help in the discussion of a concrete dynamical problem and similarly in relativistic quantum theory, one would expect dynamical laws to be simple only in terms of unbounded field operators associated with (bounded) local observables. From this suggestion of Lagrangian field theory, dynamical equations should have the form of local nonlinear equation of motion in these fields. However, powers in the field operator, viz. $A^3(x)$, cannot be dealt with naively. A well known discussion [3] of two-point functions shows that any Lorentz-covariant field is necessarily an operator-valued distribution. Ignoring this problem, one runs into the well known trouble of ultraviolet divergences. The conjectured remedy [4] for this trouble is a careful treatment of the nonlinear term as a local function obtained by delicate space time limiting procedures. This has indeed turned out to be true in certain soluble models [5]. Little is known about the formulation of such a procedure in the general case. It is in this area that one expects

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