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# Tempered Distributions in Infinitely Many Dimensions

## I. Canonical Field Operators

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Abstract. The space of testing functions for tempered distributions is characterized in an abstract way as the maximal space in a certain class of locally convex topological vector-spaces. The main characteristic of this class is stability under the differentiation and multiplication operators.

The ensuing characterization of tempered distributions may readily be generalized to the case of infinitely many dimensions, and a certain class of such generalizations is studied. The spaces of testing elements are required to be stable under the action of the canonical field operators of the quantum theory of free fields, and it is shown that extreme spaces of testing elements exist and have simple properties. In fact, the maximal space is a Montel space, and the minimal complete space is a direct sum of such spaces.

The formalism is applied to the problem of extending the canonical field operators, and a number of extension theorems are derived. In a forthcoming paper* the theory of tempered distributions in infinitely many variables will be applied to a structurally simple linear operator equation.

### 1. Introduction

Quantum theory has motivated the study of families of linear operators, which

(i) are defined in a linear space with a scalar product, and

(ii) are required to satisfy specified algebraic relations (self-adjointness, commutation relations, etc.).

The two-fold canonical family of self-adjoint operators $p_1, p_2, \ldots$ and $q_1, q_2,$ , which satisfy the canonical commutation relations

$$\text{fcP(» } p_k] = \text{fe> } q_k] = 0 \,, \quad fa, \quad q_k] = - i\, \delta_{jk} \,,$$

is perhaps the best known example.

* *Added in proof:* KRISTENSEN, P., L. MEJLBO, and E. THUE POULSEN: Tempered Distributions in Infinitely Many Dimensions. II, Displacement Operators. Math. Scand. 14, 129—150 (1964).