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Divergence of Perturbation Theory for Bosons

By

ARTHUR JAFFE

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

Abstract. Perturbation theory is studied in two dimensional space-time. There all non-derivative boson self-interactions are renormalizable and in each order of perturbation theory there are no divergences, that is all renormalizations are finite in perturbation theory. Thus the unrenormalized perturbation series may be studied and it is shown that any interaction of the general form $H_I(x) = \lambda \sum_{j=3}^{\infty} a_j \times$ $\times : \varphi(x)^j :, a_j \geqq 0$ leads to Green's functions which are not analytic in λ at $\lambda = 0$. This result holds in momentum space at a large set of points, enough to show that the Green's functions are not distributions in the momenta which are analytic in λ at $\lambda = 0$. Furthermore the proper self energy and the two-particle scattering amplitude are shown not to be analytic in λ at $\lambda = 0$ for certain momenta on or below the bare mass shell. In the course of this analysis we use the integral representations for Feynman graphs to derive a minorization of the form $|I(p_1, \ldots, p_e)| > A B^n$ for the contribution from all n^{th} order connected graphs in a theory with an interaction of the form $H_I(x) = \lambda \sum_{j=3}^{Q} a_j : \varphi(x)^j :$. Then the constants A and B depend only on the momenta p_i, and not on the structure of a particular graph.

I. Introduction

It is interesting to study perturbation theory for self interacting bosons to discover whether it can be used as a tool to prove the existence of solutions to the field equations. The problem is to expand as a power series in the coupling constant either the vacuum expectation values of the (time-ordered) Heisenberg fields, the S-matrix elements, or the kernels which occur in the numerator and denominator of the Green's functions or S-matrix elements, and then to determine whether the expansion in question defines an analytic function of the coupling constant at zero coupling. If the answer is no, then some approximation scheme more sophisticated than perturbation theory must be used to investigate solutions of the field equations.

In order to work in the interaction picture and remain completely within a Hilbert space formalism, Haag's theorem tells us that it is necessary to study an approximate theory. This follows from the fact