Commun. math. Phys. 1, 95—111 (1965)

On the Connection between the LSZ and Wightman Quantum Field Theory*

By

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Abstract. The LSZ asymptotic condition and the Yang-Feldman equations are derived in a Wightman quantum field theory on a dense set of scattering states. The Green's distributions are shown to be sufficiently regular around the energy shell to give well-defined reduction formulae for the scattering amplitudes.

I. Introduction

The analysis of the fundamental notions of relativistic quantum mechanics has been carried out in several axiomatic formalisms [1], which differ significantly in the primary notions involved.

The quantum field theory of LEHMANN, SYMANZIK, and ZIMMERMANN [2] (LSZ) is based on a complete particle interpretation. The S-matrix is assumed to be interpolated by a set of local fields satisfying a weak convergence asymptotic condition. Furthermore, one assumes that the fields are such that Green's functions can be defined and are sufficiently regular around the mass shell, in order to give sense to the reduction formulae for the scattering amplitudes. Then the main results of the LSZ theory are analyticity properties of the S-matrix elements and the many-particle structure of Green's functions.

The more general WIGHTMAN framework [3] is based on fields which are local and Lorentz covariant operator-valued tempered distributions in a Hilbert space. Under more detailed assumptions on the spectrum of the energy-momentum operator a collision theory has been developed by HAAG [4] and RUELLE [5].

In view of the particle interpretation implied by the Haag-Ruelle asymptotic condition, it is natural to investigate the connection between the LSZ and the Wightman framework. The most ambitious program would be to prove that an LSZ theory is a special Wightman theory in which the asymptotic states are complete. More modestly, we intend to show that on a dense set of collision states the LSZ asymptotic condition

* Research supported by the National Science Foundation.