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A Theorem Concerning the Positive Metric

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Abstract. It is proved that if the n-point correlation functions of a system vanish for all $n > N$ then they vanish for all $n > 2$. The theorem is valid for a wide variety of formalisms and an explicit proof is given for a Bose system with the canonical commutation relations; a proof is sketched out for a relativistic field theory of the Wightman type. The essential property used in the proof is the positive definite metric.

1. Introduction

During the last few years a great deal of effort has been applied to the analysis of the mathematical structure of relativistic field theory (see for instance [1], [2]) and a number of general results have been obtained, some of which are valid for other physical theories. One property which is assumed in many theories, but which has been scarcely analysed, is the property of the positive definiteness of the metric in Hilbert space. It is the purpose of the present note to prove one simple theorem which is essentially a direct consequence of this property alone. This theorem is concerned with the hierarchy of functions describing the correlations between various points or events of the system, the so-called truncated functions or correlation functions. Physically one would expect that the correlation between n events would be small for n large and consequently that it might be reasonable to construct models, described by a finite number of functions, by setting all the truncated functions for $n > N$ equal to zero, where N is arbitrarily large but finite. The present theorem proves that such models are inconsistent with the positive metric unless the remaining truncated functions $n > 2$ also vanish identically. Thus the class of models obtained by this physical Ansatz contains only the well known "generalized free field", which has of course been extensively applied to the many body problem and widely analysed in relativistic field theory.

In Sec. 2 of this paper we introduce several general definitions and results and show that for a system with no internal structure the above mentioned theorem is a direct consequence of a result due to MARCINKIEWICZ. In Sec. 3 this result is extended to the case of a Bose system