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On the Vacuum State in Quantum Field Theory. II

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Abstract. We want to construct, for every local irreducible quantum field theory which fulfils the spectrum condition, a new theory with the properties:

1) It is physically equivalent to the given theory (in the sense of Haag and Kastler).

2) The representation space contains a vacuum state.

3) The new theory satisfies the spectrum condition.

4) For every bounded region $\mathcal{O}$ the two representations of the algebra $\mathfrak{A}(\mathcal{O})$ are unitarily equivalent.

5) The new theory is uniquely characterized by the properties 1)—4).

I. Introduction

In the usual treatment of quantum field theory one assumes the existence of a vacuum state, i.e. of a state which is invariant under all Translations. In an earlier paper [1] this postulate was discussed. We showed the possibility of associating to every field theory another theory in which the Hilbert space also contains a vacuum state. However, we proved only the existence of a translation-invariant positive functional on the algebra generated by all local rings. We did not know at that time what properties the theory described by this functional would have. It is the aim of this note to fill this gap. The results we get are those expected intuitively from the "particle behind the moon" argument. This confirms that the vacuum assumption is only a postulate which one can add for convenience.

The frame for our investigations will be the theory of local rings of observables which has been developpped in recent years by Araki, Haag and Kastler [2]. For our purpose the existence of a unitary representation of the translation group fulfilling the spectrum condition is very essential. This implies that we cannot adopt the purely algebraic view of the theory of local ring systems. But even if the problem had a purely algebraic aspect the author would not omit the spectrum condition, for the following reasons:

From the well studied equal-time commutation relation it is known that the number of inequivalent faithful representations of a free Bose-field is at least that of the continuum. Segal [3] showed that there