On the Structure of the Algebra of Field Operators. II

By

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Abstract. The global structure analysis of local field operators is reinvestigated.

I. Introduction

In an earlier paper [1] we investigated the global structure of a given field theory. The assumptions of that paper were that the field theory is given in terms of Wightman functions which fulfil:

1) Wightman functions are tempered distributions,

2) Invariance under the inhomogeneous Lorentz group,

3) Spectrum condition,

4) Local commutativity,

5) Condition of positive definiteness.

Furthermore, there was a hidden assumption which was not explicitly mentioned, namely,

6) For real test functions $f(x)$ the canonical representation of $A(f)$ is essentially self-adjoint and the spectral resolutions of $A(f)$ and $A(g)$ commute if the supports of f and g are spacelike to one another.

In another paper with ZIMMERMANN [2] we studied the question whether 6) can be derived from the assumptions 1)—5). It turned out that 6) can only be derived if certain very restrictive conditions are fulfilled. We gave a trivial counterexample showing that 6) is not a consequence of the other assumptions. The additional condition we had to introduce is so restrictive that even the Wick-ordered cube of a free field does not satisfy it. WIGHTMAN [3] gave arguments indicating that this field is indeed not self-adjoint. In this special case one can construct ring systems, and JAFFE [4] has shown that they are local.

Recently ARAKI [5] investigated the global structure of local ring systems and found that such a theory is only irreducible if the Hilbert space contains a single vacuum state when cyclicity is assumed.