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The C*-Algebras of a Free Boson Field

I. Discussion of the Basic Facts

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Abstract. We give a systematic description of several C^*-algebras associated with a free Boson field. In this first part the structure of the one-particle space enters only through its symplectic form σ and a directed absorbing set of finite-dimensional subspaces on which σ is non-degenerate. The Banach *-algebras $\mathscr{L}_1(\mathfrak{E}, \sigma)$ and $\mathscr{M}_1(\mathfrak{E}, \sigma)$ of absolutely continuous resp. bounded measures on a finite-dimensional symplectic space $(\mathfrak{E}, \sigma)$, with their "twisted convolution product" stemming from Weyl's commutation relations, are studied as the analogues of the $\mathscr{L}_1$ resp. $\mathscr{M}_1$ algebras of a locally compact group. The fundamental "vacuum idempotent" Ω determines their (unique) Schrödinger representation, Schrödinger A^*-norm and Schrödinger C^*-completions $\overline{\mathscr{L}_1(\mathfrak{E}, \sigma)}$ and $\overline{\mathscr{M}_1(\mathfrak{E}, \sigma)}$. After a study of these one proceeds to a construction as an inductive limit of the algebras $\mathscr{M}_1(\mathfrak{H}, \sigma)$ and $\overline{\mathscr{M}_1(\mathfrak{H}, \sigma)}$ for an infinite-dimensional symplectic space $(\mathfrak{H}, \sigma)$. The "Fock representations" (with the corresponding "field operators") are presented as the infinite-dimensional generalization of the Schrödinger representation. The paper ends with a discussion of several possible choices for the "free Boson C^*-algebra".

§ 1. Introduction

The present paper is the first part of a study of different C^*-algebras associated with the free Bose field. Our aim is to investigate on the example of the free relativistic Bose field a number of questions which arise in HAAG's approach to field theory based on "local rings" [1—5] — particularly in the version of this approach based on C^*-algebras [6, 7]. Some of the questions we have in view are the following:

1) Amongst the different, more or less "rich" C^*-algebras which can be associated with the relativistic free Bose field, which one should be chosen as the "quasi-local algebra"?

2) What is the relation between the space-time structure and the algebraic structure ("diamond theorem" . . . etc.)?

3) Does HAAG's conjecture that local factors are of Type I [8] hold for some adequately chosen faithful representation of the quasi-local algebra (it has been shown not to hold for the standard Fock representation by ARAKI [9])? This question leads to formulating the following conjecture: for adequately chosen "commuting" space-time