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Consistency of Relativistic Particle Theories

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Abstract. While direct-interaction particle theories are generally thought to be incompatible with relativity in classical physics, such relativistic theories in quantum mechanics have recently attracted attention. The reasons for rejecting these theories in classical physics are based on the consideration of world lines, while relativistic quantum mechanics has no covariant position operator so that the classical refuting argument cannot be adapted.

This paper discusses the consistency of relativistic particle theories with a finite number of degrees of freedom, without recourse to the position operator. A particle is described by a sub-algebra of observables at one time. Homogeneous transformations, including "accelerations," must preserve the identity of particles, and therefore leave the sub-algebras invariant. It is shown that with this assumption only non-interacting particle theories are compatible with the principle of relativity, in quantum as well as classical mechanics.

I. Introduction

Relativistic quantum mechanical particle theories [1—4] — as contrasted to field theories — have recently attracted great attention, because they make phenomenological two- or three-particle problems amenable to a rigorously relativistic, mathematically consistent treatment. On the basis of classical considerations, it had long been believed that the relativistic interaction of two particles required a field, i.e. infinitely many variables. Hence, the new developments are somewhat surprising. Is it possible that quantum mechanics can succeed in a direct-interaction relativistic theory where classical physics fails, and if so, what are the virtues of quantum mechanics which make it possible ?

Closer inspection shows that the success of quantum mechanical particle theories is bought for a price: the particle variables q_i are the Wigner-Newton position operators which do not "transform covariantly" and therefore cannot be seriously considered as the quantum mechanical equivalent of the classical position variable x_μ ($\mu = 1$ to 3). This circumstance raises a question of principle, both in classical and in quantum mechanics: is the instantaneous position of a particle a necessary and

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