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# An Algebraic Spectrum Condition

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**Abstract.** It is shown that a translationally invariant algebra $\mathfrak{A}$ of local observables (see [1]) admits a representation in a Hilbert space having a vacuum state. Furthermore an algebraic criterion is given which is necessary and sufficient for the existence of at least one representation of $\mathfrak{A}$ in which the usual spectral condition for the energy-momentum operators holds.

## I. Introduction

A local ring system [1] is defined by giving a $B^*$- algebra* on which a space-time translation $x$ operates as a *-automorphism

$$A \to A_x\,, \quad A \in \mathfrak{A}\,, \tag{1}$$

and by assigning to each space-time domain $\mathcal{O}$ a Banach *-subalgebra $\mathfrak{A}(\mathcal{O})$ of $\mathfrak{A}$ such that

$$\mathfrak{A}(\mathcal{O}_1) \text{ commutes with } \mathfrak{A}(\mathcal{O}_2) \text{ if } \mathcal{O}_1 \text{ is spacelike to } \mathcal{O}_2\,, \tag{2a}$$

$$[\mathfrak{A}(\mathcal{O})]_x = \mathfrak{A}(\mathcal{O} + x)\,, \tag{2b}$$

$$\text{if } \cup\, \mathcal{O}_n = \mathcal{O},\ \cup\, \mathfrak{A}(\mathcal{O}_n) \text{ generates } \mathfrak{A}(\mathcal{O});\ \overline{\cup\, \mathfrak{A}(\mathcal{O})} = \mathfrak{A}. \tag{2c}$$

The important work of Borchers, Haag and Schroer [2] showed that if there exists a Hilbert space representation of a local ring system in which the transformations (1) are induced by a unitary representation of the translation group

$$A_x = U(x)\, A\, U(x)^{-1}\,, \tag{3}$$

then there also exists such a representation with a vacuum state:

$$U(x)\, \Psi_0 = \Psi_0\,. \tag{3'}$$

This representation satisfies the spectrum condition

$$P^2 > 0, \quad P_0 > 0 \qquad (\text{where } \exp[i P_\mu x^\mu] = U(x)) \tag{4}$$

if the initial one did [3].

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* Terminology of C. E. Rickart, *General Theory of Banach Algebras*. Naimark [4] uses the term "completely regular Banach ring".