COMMUNICATIONS IN INFORMATION AND SYSTEMS
Vol. 8, No. 3, pp. 223-244, 2008

003

QUANTIZATION WITH ADAPTATION - ESTIMATION OF GAUSSIAN LINEAR MODELS*

LÁSZLÓ GERENCSÉR†, ILDIKÓ KMECS‡, AND BALÁZS TORMA§

Abstract. Quantization is a basic operation in communication, having a considerable impact also on control, in particular on control over communication networks, see [2] for an early reference. In this paper we consider a classic, seemingly innocent problem of reconstructing a single signal value θ^* when measured with additive Gaussian noise, followed by uniform quantization of sensitivity h, with or without saturation. A peculiar feature of the above estimation problem is that its Fisher information varies considerably with the noise variance and the location of the true parameter. It is therefore a meaningful objective to adjust (shift) the quantization levels so as to maximize the Fisher information or to inject additional measurement noise for the same purpose. We shall focus on the first problem. Empirical evidence shows that, for given noise variance, the Fisher information is maximal when the location parameter is of the form $\theta^* = kh+h/2$. Adjusting the quantization levels is equivalent, from the statistical point of view, to adjusting, say increasing the location parameter by an amount of $\delta > 0$ to achieve a *known* target, say $\eta^* = kh+h/2$ for some integer k. The problem that we address in this paper is if such an adjustment of the problem can be done adaptively, in the context of a previously developed recursive, real-time estimation method for estimating θ^*, that was called a randomized EM-method for estimating θ^*. We give a positive answer to this question. The proposed method results in considerable improvement in efficiency, supported both by the algebra of the asymptotic theory of stochastic approximation, and by extensive experimental evidence. The basic ideas developed and presented for this benchmark problem can be easily generalized for the multi-variable case.

Keywords: quantization; Gaussian linear regression; EM-method; Metropolis-Hastings method; stochastic approximation.

1. Introduction.

Quantization is a basic operation in communications, arising among others in analog-to-digital conversion or in data-compression, having a considerable impact also on control, in particular on control over communication networks. The latter application area is particularly fit for the present issue honoring Roger Brockett, due to his fundamental contribution to the area in his 1998 paper, (co-authored by D. Liberzon), [2]. See also [14] for a recent survey on quantization in communication and control.

A scalar quantizer is defined as a mapping q from $\mathbb{R}$ to a discrete, finite or countable set $\mathcal{Y} \subset \mathbb{R}$, representing the so-called quantization levels, assigning to each

*Dedicated to Roger Brockett on the occasion of his 70th birthday.

†Corresponding author, MTA SZTAKI, 13-17 Kende u., Budapest 1111, Hungary.
E-mail: gerencser@sztaki.hu, Tel: (36-1)-279-6138, (36-1)-279-6190, Fax: (36-1)-466-7503.
‡SIEMENS PSE KFT, 51-57 Gizella út, Budapest 1143, Hungary.
E-mail: ildiko.kmecs@siemens.com, Tel: (36-1)-471-3576.
§MTA SZTAKI, 13-17 Kende u., Budapest 1111, Hungary.
E-mail: torma@sztaki.hu, Tel: (36-1)-279-6158.