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ON AN EIGENFLOW EQUATION AND ITS LIE ALGEBRAIC GENERALIZATION*

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Abstract. This paper deals with a dynamical system of the form $\dot{A} = [[N, A^T + A], A] + \nu[[A^T, A], A]$, where A is an $n \times n$ real matrix, N is a constant $n \times n$ real matrix, ν is a positive constant and $[A, B] = AB - BA$. In particular, the purpose of this paper is to establish a sorting behavior of the dynamical system and to represent it in a general Lie algebraic setting. Moreover, some applications of the dynamical system are presented.

Keywords: Control theory, dynamical systems, ordinary differential equations

1. Introduction.

Brockett introduced in [6] the so-called double-bracket equation

$$\dot{H} = [[N, H], H], \tag{1.1}$$

where H is an $n \times n$ real symmetric matrix, N is a constant $n \times n$ real symmetric matrix, and $[A, B] = AB - BA$. This dynamical system has several remarkable properties. For example (1.1) can be used to sort lists or to diagonalize symmetric matrices. In the recent paper [13], a dynamical system of the form

$$\dot{A} = [[N, A^T + A], A] + \nu[[A^T, A], A], \tag{1.2}$$

where A is an $n \times n$ real matrix, N is a constant $n \times n$ real matrix, and ν is a positive constant, has been introduced. The dynamical system (1.2) has similar properties as (1.1), as will be shown in this paper, but it also diagonalizes and computes eigenvalues of *nonsymmetric* matrices. In the case of A being symmetric, the self-commutator $[A^T, A]$ vanishes and (1.2) reduces to (1.1). Thus, the flow in the space of nonsymmetric matrices described by (1.2) can be considered as a generalization of the flow in the space of symmetric matrices described by (1.1). Due to the ability to simultaneously compute all eigenvalues of nonsymmetric matrices, one may call (1.2) eigenflow equation.

The motivation to design and study dynamical systems like (1.1) and (1.2) has several roots. For example, solving computational problems with the help of continuous-

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