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SPECIAL SOLUTIONS TO SOME KOLMOGOROV EQUATIONS ARISING FROM CUBIC SENSOR PROBLEMS*

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1. Introduction. Ever since the technique of the Kalman-Bucy filter was popularized, there has been an intense interest in developing nonlinear filtering theory. Basically we have a signal or state process $x = \{x_t\}$ which is usually not observable. What we can observe is a related process $y = \{y_t\}$. The goal of nonlinear filtering is to determine the conditional expectation of the form $E[\phi(x_t) : y_s, 0 \leq s \leq t]$ where ϕ is any C^∞ function or even better to compute the entire conditional probability density $\rho(t, x)$ of x_t given the observation history $\{y_s : 0 \leq s \leq t\}$. In practical applications, it is preferable that the computation of conditional probability density be preformed recursively in terms of a statistic $\theta = \{\theta_t\}$, which can be updated by using only the latest observations.

In some cases, θ_t is computable with a finite system of differential equations driven by y. This leads to the ideal notion of finite dimensional recursive filter. By definition such a filter is a system:

$$d\theta_t = \alpha(\theta_t)dt + \sum_{i=1}^{p} \beta_i(\theta_t)dy_{it}$$

driven by the observation y_{it} where y_{it} is the i-th component of y_t, $i = 1, \cdots, p$; together with an output map

$$E[\phi(x_t) : y_s, 0 \leq s \leq t] = \gamma(\theta_t).$$

In the 1960s and early 1970s, the basic approach to nonlinear filtering theory was via "innovation methods" originally proposed by Kailath [Ka] and Frost and Kailath [Fr-Ka] and subsequently rigorous developed by Fujisaki, Kallianpur, and Kunita [F-K-K] in 1972. As pointed out by Mitter [Mi], the difficulty with this approach is that innovation process is not, in general, explicitly computable (except

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