could prove its own consistency." Since $\prec$ is arithmetically definable, Gentzen's induction principle can be expressed as a "schema" in the language of **PA** in the same manner as ordinary induction. In the system obtained by adding this principle to **PA** (call it **PA+**), Gentzen's consistency proof for **PA** can certainly be carried out. But this is not an instance of a "theory which could prove its own consistency;" the consistency of **PA** is proved in a different system **PA+**. There is also a misstatement on p. 215: the authors surely meant to say that it *was* clear to Gödel that the primitive recursive functions were *not* "all the computable ones... ."

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Linear operators in spaces with an indefinite metric, by T. Ya. Azizov and I. S. Iokhvidov. John Wiley & Sons, 1989, 300 pp., $82.95. ISBN 0-471-92129-7

Let H be a Hilbert space (over the complex numbers), and let J be a bounded linear selfadjoint operator on H such that $J^2 = I$. Consider the sesquilinear form $[\cdot, \cdot]$ induced by J:

$$[x, y] = \langle Jx, y\rangle, \qquad x, y \in H,$$

where $\langle\cdot, \cdot\rangle$ stands for the scalar product in H. The corresponding quadratic form $[x, x]$ is indefinite (unless $J = I$ or $J = -I$), in other words, there exist $x, y \in H$ for which $[x, x] < 0$ and $[y, y] > 0$. The space H, together with the sesquilinear form $[\cdot, \cdot]$ generated by some J as above, is commonly called a *Krein space.* One can also define the Krein spaces intrinsically, by starting with a topological vector space and a continuous sesquilinear form on it, and by imposing suitable completeness and nondegeneracy axioms. The reviewed book is devoted to the geometry of Krein spaces and the spectral structure and related properties of several important classes of bounded and unbounded linear operators on Krein spaces.

1. THE SUBJECT

Why Krein spaces? As with many mathematical disciplines, there are two compelling reasons: (1) important applications in